Phononics

2011

1st International Conference on Phononic Crystals, Metamaterials & Optomechanics

May 29-June 2 2011, Santa Fe, New Mexico, USA

Felix Bloch Lecture

Felix Bloch (1905-1983)
Nobel Prize in Physics (1952)

2011 Felix Bloch Lecturer: Professor Sia Nemat-Nasser
Director, Center of Excellence for Advanced Materials
Distinguished Professor of Mechanics and Materials
University of California, San Diego, USA

Felix Bloch Lecture

The Felix Bloch Lecture honors the eminent Swiss physicist who among many contributions to wave mechanics and theoretical physics formulated the underlying theory for electron wave propagation in periodic media. His theory, known as *Bloch theory*, laid the foundation for other theoretical developments ultimately leading to a formal classification of all crystals into metals, semiconductors and insulators. In recent years, Bloch theory re-emerged as the basic underlying mathematical condition for formulating the band structure of modern periodic materials such as phononic and photonic crystals. Felix Bloch, who was born in Zurich, Switzerland, on October 23, 1905, pursued his research career in Zurich, Heisenberg, Stanford, Los Alamos and Harvard University, and in 1954 took a leave of absence for one year to serve as the first Director General of CERN in Geneva. He received the Nobel Prize in Physics in 1952 jointly with Edward Mills Purcell "for their development of new methods for nuclear magnetic precision measurements and discoveries in connection therewith".

In Phononics 2011, the *Felix Bloch Lecture* is being inaugurated to honor researchers who have made outstanding contributions to the field of *phononics*.

The 2011 Felix Bloch Lecturer is **Professor Sia Nemat-Nasser** who is honored for his invaluable contributions to the area of "wave propagation in elastic periodic media" starting with the paper that discusses "Generalized variational methods for waves in elastic composites" published in 1972 through his recent work on homogenization and the determination of dynamic effective properties.

2011 Felix Bloch Lecture Committee

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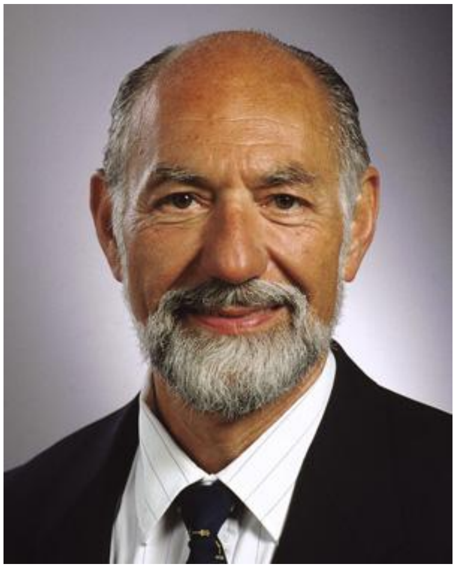

Professor Sia Nemat-Nasser

Sia Nemat-Nasser is the Director of the Center of Excellence for Advanced Materials, and Distinguished Professor of Mechanics and Materials, University of California, San Diego. He arrived in the USA in 1958 from Iran. His *EDUCATION* includes: A B.S. in engineering from California State University, Sacramento (1960), M.S. in civil engineering (1961) and Ph.D. in engineering (1964) from UC Berkeley. His *ACADEMIC CAREER* spans*:* Assistant professor, Sacramento State University, 1961-62. Post-doctoral researcher, 1964-66, Northwestern University; assistant and associate professor, 1966-70, University of California, San Diego; professor, 1970-85, Northwestern University; professor, 1985-present, University of California, San Diego. He spearheaded the creation of an integrated Materials Science and Engineering Graduate Program, serving as its Founding Director (1989-1994). He was Co-Director of NSF's Institute for Mechanics and Materials (1992-1997) and Director (1997-1999). *AWARDS AND HONORS* include: *Nat'l. Acad. Eng.:* Member 2001. *ASME:* Stephen P. Timoshenko Medal 2008; Mater. Div. establishes the Sia Nemat-Nasser Early Career Medal 2008 (focused on underrepresented minorities & women in engineering); Robert H. Thurston Lecture Award 2006; Honorary Member 2005; Aerospace Division, Adaptive Structures and Material Systems Best Paper of the Year Award 2003; Nadai Medal 2002; Life Fellow 2001; Chair, Mater. Div. 1997-98; Fellow 1979. *ASCE:* Theodore von Karman Medal 2008. *Sacramento State University:* Distinguished Alumni Award 2008. *SEM:* establishes the Sia Nemat-Nasser Award (2009) with Sia as its first recipient (2011); SEM Fellow 2011; W.M. Murray Medal 2009; B.J. Lazan Award 2007. *SES:* William Prager Medal 2002; Fellow 1988; Pres. 1979-80. *Amer. Acad. Mech.:* Pres. 1996-97, Secy. 1989-94, Founding Fellow 1970. *World Innovation Foundation:* Honorary Member 2004. *Tech. Inst.:* Willard F. Rockwell Medal 2003. *UC Berkeley:* Appointed Russell Severance Springer Professor – Fall Semester 2010. *UCSD:* Faculty Research Lecturer Award 2005; Sch. Eng./MAE Teacher of the Year Award, 2000-01, 1996-97, & 1994-95; John Dove Isaacs Chair in Natural Philosophy 1995-00. Alburz Educational Foundation Prize 1975.

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PHONONICS-2011-BlochLecture

Negative Effective Dynamic Mass-density and Stiffness: Micro-architecture and Phononic Transport in Periodic Composites

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Abstract: I am humbled to be given the honor to present a lecture named for a great physicist, Felix Bloch. I will focus on real structural composite materials that in addition to their usual static attributes, such as strength and ductility, exhibit unusual metamaterial-type dynamic responses. At subwavelength, the micro-architecture of these composites can be designed to provide them with controlled stop-bands, pass-bands, negative and positive dispersive characteristics, and, most remarkably, negative effective dynamic mass density and stiffness. I will examine both electromagnetic and acoustic properties of these materials.

Introduction

Waves in Elastic Composite Materials: In 1956, Rytov studied[1] the Bloch-form[2] or Floquet-type[3] elastic waves propagating normal to layers in a periodic layered composite and produced the expression for the dispersion relation that gives the pass-bands and stop-bands in the frequency-wave number space. Because of the emergence of structural composite materials with application to aerospace and other technologies, the 1960's witnessed considerable scientific activity mostly focused on estimating the effective static properties of composites, whereby elegant and rigorous bounds for their effective properties have been established; Hashin and Shtrikman [4], Hill[5], Nemat-Nasser *et al.*[6], Willis[7], Nemat-Nasser and Hori[8, 9].

The early effort to study the dynamic response of elastic composite materials was mostly limited to one-dimensional problems. Because composites are generally anisotropic, they admit various longitudinal and shear waves which complicate the corresponding analysis; the elasticity tensor, for example, admits six eigenvalues, each associated with a second-order symmetric eigenstrain field. To create a general numerical approach to solving elastic waves in composites, Kohn et al. proposed[10] using a modified version of the Rayleigh quotient in conjunction with the Bloch-form waves to calculate the dispersion curves. To directly account for the strong discontinuities that generally exist in the elastic properties of the composites' constituents, Nemat-Nasser developed[11] a mixed variational formulation to calculate the eigenfrequencies and modeshapes of harmonic waves in 1-, 2-, and 3-dimensional periodic composites. The approach is based on a general variational method in which the stress and displacement are varied independently. The efficacy and accuracy of the approximate method were demonstrated by means of numerical examples (See Nemat-Nasser[11], Nemat-Nasser *et. al.*[12], Minagawa and Nemat-Nasser[13]) using Fourier series approximation. Babuska and Osborn subsequently proved[14] the convergence of the method. They showed that while the rate of convergence of both the Rayleigh quotient and the mixed formulation depends upon the regularity of the elastic properties and the density through the unit cell, the mixed formulation always has a convergence rate faster than that of the Rayleigh quotient. The general method therefore is a powerful tool for effectively calculating the band structure for structural composite materials.

Electromagnetic Waves in Periodically Distributed Conductors; Meta-materials: In his paper on hypothetical materials, Veselago theoretically investigated[15] the electromagnetic consequences of having a material with simultaneously negative electrical permittivity (ε) and magnetic permeability (μ). He surmised that such a material would exhibit interesting propagation characteristics such as reversed Doppler shift and Cherenkov radiation, and anomalous refraction. Since refractive index is given by $n = \sqrt{\varepsilon\mu}$ where both ε and μ are generally positive in naturally occurring materials, materials with simultaneously negative ε and μ also give rise to a real refractive index (taken as negative). Such materials with negative index of refraction exhibit phase velocity which is anti-parallel to the group velocity. Naturally occurring materials do display negative electrical permittivity (silver, gold at optical frequencies) and negative magnetic permeability (resonant ferromagnetic and antiferromagnetic systems) but the frequency ranges where such properties are exhibited do not overlap, thereby precluding the possibility of wave transmission. Even in naturally occurring materials, negative ε and μ are a result of subwavelength resonances in the electric and magnetic fields. Since there is no natural law which prohibits the overlap of the frequencies where negative ε and μ are exhibited by a material, it is conceivable to construct artificial materials with subwavelength resonances tuned such that they give rise to effective negative ε and μ in a common frequency range. Such materials where the subwavelength microstructure results in the material exhibiting unusual effective properties have come to be called *metamaterials*. Veselago's hypothetical material with simultaneously negative ε and μ was finally realized by Smith *et. al.*[16], using a periodic structure consisting of thin copper wires and thin split-ring resonators, as has been theoretically considered by Nicorovici and McPhedran[17], and Pendry *et. al.*[18] Finally, using a similar structure in a wedge shape, Shelby *et. al.* verified[19] the existence of the negative index of refraction, and shortly after, structural composite materials with tuned turn-on frequency (Fig. 1, right) and with negative index of refraction were designed, fabricated (Fig. 1, left), and characterized (Fig. 1, middle); Nemat-Nasser *et al.*[20] and Starr *et al.*[21]

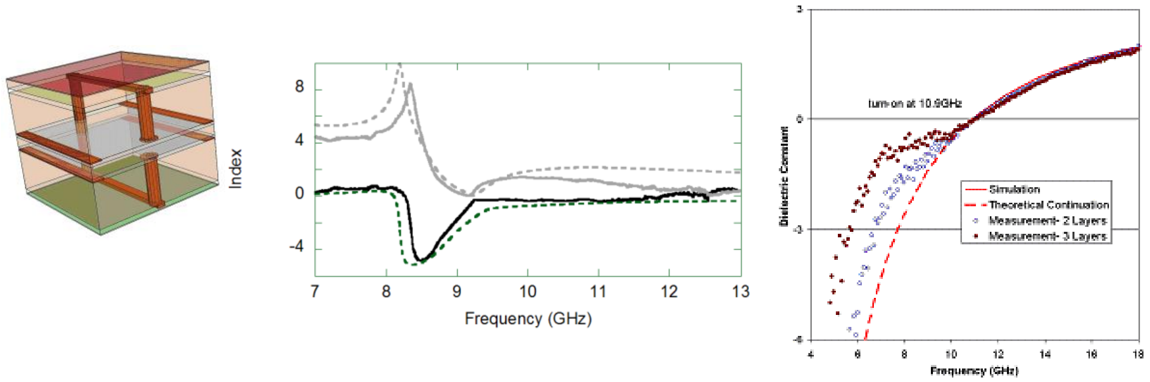


Figure 1 (right): Numerical and experimental characterization of Cyanate Ester/Quartz composite with embedded 50μm thin-wires and an EM turn-on frequency of 10GHz; **(middle):** Recovered refractive index (n) from simulation data (dashed curves) and from measured S-parameters (solid curves); black and gray curves represent the real and imaginary parts of the refractive index, respectively; **(left):** The unit cell of negative-index composite.

The theoretical and experimental successes of electromagnetic metamaterials suggest that it may be possible to study and design the acoustic analogues of electromagnetic metamaterials. Electromagnetic metamaterials have been realized by designing a periodic subwavelength microstructure which has a desired resonant behavior. The effective properties are defined by relating certain line and surface averages within a unit cell to the electric and the magnetic fields. The analogous acoustic problem of wave propagation in periodic composites is complicated by the existence of both longitudinal and shear modes. Furthermore, while both effective magnetic permeability and electrical permittivity are second-order tensors, in acoustics the effective dynamic elasticity tensor is fourth-order and density is a second-order tensor, see Willis[22]. This indicates that the characterization of the effective overall properties for phononic transport in a periodic composite is complicated, as has been discussed[23] by Hussein; see also references cited therein. Therefore, to be able to design acoustic metamaterials with desired effective properties, the following twin problems need to be tackled:

1. Calculation of band-structure of periodic composites in 1-, 2-, and 3-dimensions.
2. A consistent homogenization procedure that uses the eigenvalue/modeshape information from step 1 and provides a set of effective constitutive relations relating the unit cell averages of stress, strain, velocity, and momentum.

One-dimensional periodic layered composite: Consider a 1-D periodic layered composite (Fig. 2, right) with a plane harmonic longitudinal stress wave traveling perpendicular to the layers. The first four modes are compared in Fig. (2, left). It can be seen that the mixed method gives accurate results for the first three modes when $Mp = 2*M+1=5$ terms are used in the Fourier expansion to approximate the displacement and stress. The fourth mode is inaccurate for the $Mp = 5$ calculation but as the number of terms in the expansion is increased to $Mp = 7$ $(M = 3)$, the results converge to those obtained from the exact solution.

Since the exact dispersion relations are available for only fairly simple geometries like layered composites, the mixed variational formulation provides an attractive and effective method to calculate the eigenfrequencies and eigenvectors associated with 1-, 2-, and 3-D periodic composites.

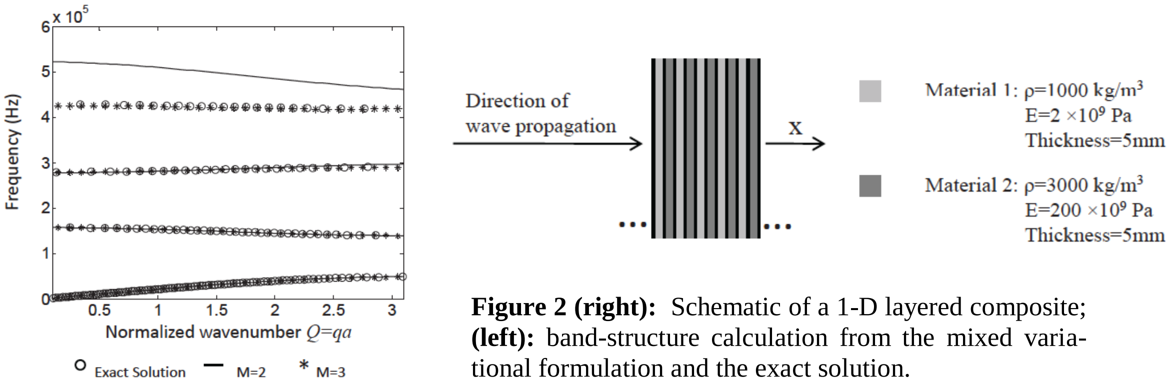


Figure 2 (right): Schematic of a 1-D layered composite; **(left):** band-structure calculation from the mixed variational formulation and the exact solution.

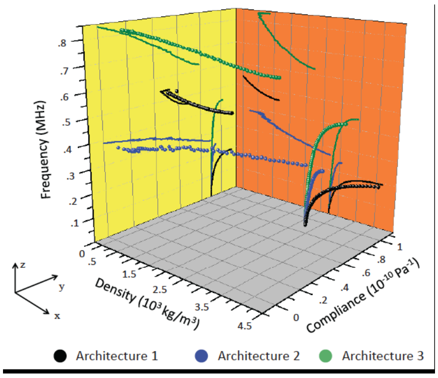
Defining effective dynamic parameters: Homogenization

The elastostatic response of composites is long understood to be non-local in space, Beran[24], Nemat-Nasser *et al.*[6], Willis[7], Diener *et. al.*[25], and Bakhvalov and Panasenko[26]; see Nemat-Nasser and Hori[8] for a comprehensive account. In the context of inhomogeneous elastodynamics, effective constitutive relations are non-local in both space and time. Additionally, average stress is coupled both with average strain and average velocity, and average momentum is coupled not only with average velocity but also with average strain, see Willis[22,27-29], and Shuvalov *et. al.*[30] The structure of the averaged constitutive law for a general 3-D composite is given by,

$$\langle \boldsymbol{\varepsilon} \rangle = \boldsymbol{D}^{eff}(\omega, \boldsymbol{q}) : \langle \boldsymbol{\sigma} \rangle + \boldsymbol{S}^{1}(\omega, \boldsymbol{q}) . \langle \dot{\boldsymbol{u}} \rangle$$

$$\langle \boldsymbol{p} \rangle = \boldsymbol{S}^{2}(\omega, \boldsymbol{q}) : \langle \boldsymbol{\sigma} \rangle + \boldsymbol{\rho}^{eff}(\omega, \boldsymbol{q}) . \langle \dot{\boldsymbol{u}} \rangle$$

where $\boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \boldsymbol{p}, \dot{\boldsymbol{u}}$ represent the field variables strain, stress, momentum and velocity respectively. $\boldsymbol{D}^{eff}$ is the fourth-order effective compliance tensor with minor symmetries, $D^{eff}_{ijkl} = D^{eff}_{jikl} = D^{eff}_{ijlk}$, and hermitian symmetry, $D^{eff}_{ijkl} = [D^{eff}_{klij}]^{*}$, where * denotes conjugation. Effective density is a second-order hermitian tensor, $\rho^{eff}_{ij} = [\rho^{eff}_{ji}]^{*}$, (see Willis[31]), and the coupling tensors, $\boldsymbol{S}^1$ *and* $\boldsymbol{S}^2$, are third-order with $S^1_{ijk}=[S^2_{kij}]^{*}$; for proofs and details see Nemat-Nasser and Srivastava[32], Srivastava and Nemat-Nasser[33]. The above structure of the constitutive relations is equivalent to the self-adjoint constitutive structure proposed by Milton and Willis[34].



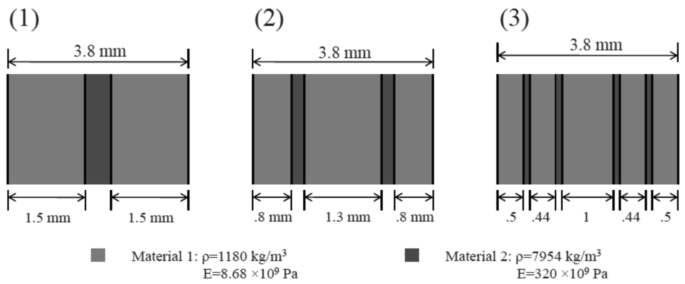


Figure 3 (right): Three micro-architectures that use the same amount of each constituent; **(left):** resulting effective dynamic density and compliance as functions of frequency.

The time-space nonlocal character of these relations, displayed by their dependence on ω *and* $\boldsymbol{q}$, can be exploited to design composites with novel dynamic properties, using the same amount of each constituent per unit volume. For a layered composite, this is displayed in Fig. 3 in terms of a *dynamic Ashby Chart*; see Nemat-Nasser et al.[35]

Locally resonant phononic metamaterials

The actual calculation of the effective dynamic parameters for periodic composites has been of great interest during the last decade. As a minimum, one must ensure that the resulting effective parameters satisfy: (1) the overall conservation laws and compatibility relations; and (2) the composite's dispersion relations.

For Bloch-type electromagnetic waves in periodic media, a direct homogenization technique, based on micromechanical principles, was proposed[36] by Amirkhizi and Nemat-Nasser. Their results do satisfy the above-stated two basic requirements. Willis provided[37] an ensemble averaging-based homogenization method whose results were shown to be equivalent to the field integral method given in Nemat-Nasser *et. al.*[35] Subsequent work in dynamic homogenization using micromechanics has provided explicit tools to calculate the effective dynamic properties of periodic elastic composites with *any microstructure*; Nemat-Nasser and Srivastava[32], and Srivastava and Nemat-Nasser[33].

These and related theoretical estimates suggest that unusual dynamic responses can be induced in composites by designing locally resonating microstructures, Sheng *et. al.*[38], Liu *et. al.*[39], Avila *et. al.*[40], and Milton and Willis[34]. The effective dynamic density and effective dynamic compliance can be designed to be negative over specific ranges of frequencies. This is illustrated in Fig. 4, using a 5-layered composite which has a locally resonating central heavy and stiff layer (Material 3) sandwiched between two compliant layers (Material 2). It can be seen from Fig. 4 that this composite clearly exhibits negative effective density and stiffness in the frequency range *~23-35 kHz*. In fact, it is seen in the calculations that as Material 2 is made stiffer, the frequency range within which the effective parameters are simultaneously negative becomes progressively smaller until it disappears.

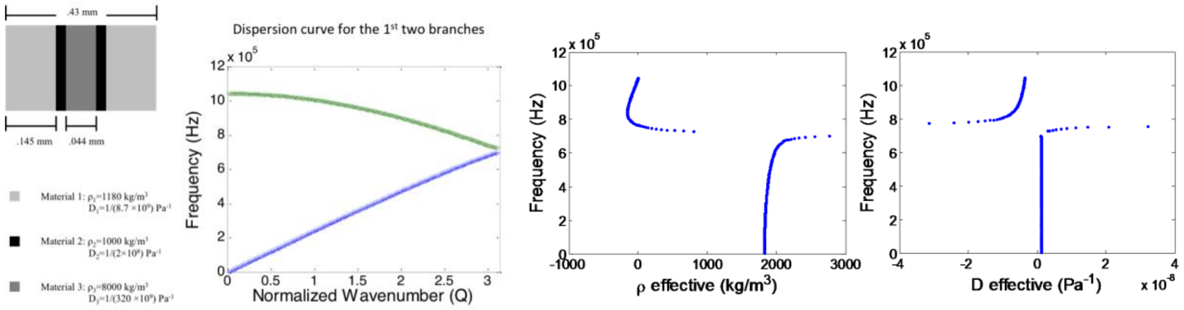


Figure 4 (right): Effective dynamic mass-density and compliance of a 5-layered composite; **(left):** geometry of the 5 layers and the dispersion curves (calculation by Dr. Ankit Srivastava).

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